SECTION—D

7. (a) If X and Y are independent Gamma variate with parameters μ and ν respectively, then show that $u = X + Y$ is also distributed as gamma with parameter $(\mu + \nu)$.

(b) What is a normal distribution ? Describe its important properties.

8. (a) What are the requirements of a measure of association of two attributes ? State two coefficients of association of attributes. Show that they lie between –1 and 1.

(b) Given that :

Regression equations :

$8x - 10y + 66 = 0$ and

$40x - 18y = 214$

Variance of x : 9

Find :

(i) Mean values of x and y

(ii) The correlation coefficient between x and y

(iii) Standard deviation of y.

Exam. Code : 211003
Subject Code : 4970

M.Sc. Mathematics 3rd Semester (Batch 2020-22)

MATH-577 : STATISTICS—I

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each Section. The **fifth** question may be attempted from any Section. All questions carry equal marks.

SECTION—A

1. (a) Define various measures of dispersion. Prove that standard deviation is minimum value of root mean square deviation.

(b) For three events A, B and C in a sample space, show that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) - P(A \cap B \cap C)$.

2. (a) Give various approaches to define probability. Write limitations of each, if any.

(b) A bag contains three coins, one of which is coined with two heads while the other two coins are normal and not biased. A coin is chosen at random and tossed four times in succession. If head turns up each time, what is the probability that it is the two headed coin ?

SECTION—B

3. (a) Let the joint probability density function of the random variables X and Y be :

$$f(x, y) = \begin{cases} 2(x+y-3xy^2) & ; \quad 0<x<1, 0<y<1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Find :

(i) Marginal distributions of X and Y

(ii) E(X+Y) and E(X–Y)

(b) Let X be a arbitrary random variable with distribution function F, show that :

$$E(X) = \int_0^\infty [1-F(x)]dx - \int_{-\infty}^0 F(x)dx$$

4. (a) Let (X, Y) be a two-dimensional non-negative continuous random variable having the joint density function :

$$f(x, y) = \begin{cases} uxy\, e^{-(x^2+y^2)} & ; x \geq 0, y \geq 0 \\ 0 & ; \text{otherwise} \end{cases}$$

Find the distribution of $U = \sqrt{X^2+Y^2}$

(b) The joint probability density function of random variable X and Y is given by :

$$f(x, y) = \begin{cases} 8xy & ; 0<x<y<1 \\ 0 & ; \text{otherwise} \end{cases}$$

Find :

(i) E(Y/X = x)

(ii) V(Y/X = x).

SECTION—C

5. (a) A random variable X has the distribution with prob. density function $f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}; x > 0, \theta > 0.$

Use Chebyshev's inequality to obtain a lower bound to the probability $P(0 < \overline{x} < 2\theta)$ and compare it with the actual probability.

(b) Define Poisson distribution. Obtain the recurrence relation of it for central moments.

6. (a) Let $X_1, X_2, ...$ be independent and identically distributed Poisson variates with parameter λ. Use CLT to estimate $P(120 \leq S_n \leq 160)$,

where $S_n = \sum_{i=1}^{n} X_i$; λ = 2 and n = 75

(b) Define geometric distribution of a random variable. Obtain its mean and variance. Also prove the lack of memory property of it.